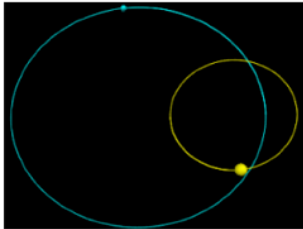


September 22

Get Clickers

VPython Exoplanet



[NASA site](#) and [exoplanets.org](#)

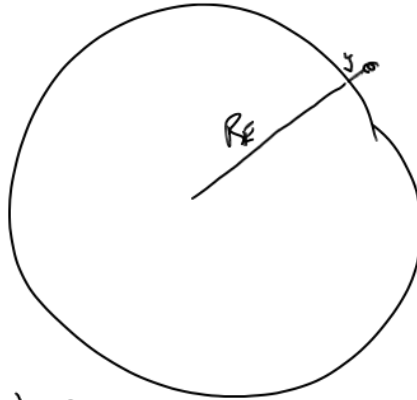
[Doppler Effect](#)

Gravity near the Earth's surface

$$\vec{F}_{\text{gravity on 2 by 1}} = -G \frac{m_1 m_2}{|\vec{r}|^2} \hat{r}, \quad \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{F}_{\text{grav}}| = \frac{G m_1 m_2}{|\vec{r}|^2}$$

$$|\vec{F}_{\text{near Earth}}| = \frac{G M_{\oplus} m}{(R_{\oplus} + y)^2}$$



binomial expansion

$$(1+a)^n = 1 + na + \frac{n(n-1)}{2}a^2 + \dots$$

for $a \ll 1$, only need first few terms

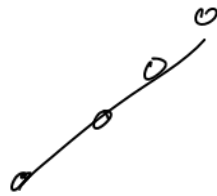
$$|\vec{F}_{\text{near surface}}| = \frac{G M_{\oplus} m}{R_{\oplus}^2 (1 + y/R_{\oplus})^2} = \frac{G M_{\oplus} m}{R_{\oplus}^2} \left(1 - \frac{2y}{R_{\oplus}} + \dots \right) \approx \frac{G M_{\oplus}}{R_{\oplus}^2} m \left(1 - \frac{2y}{R_{\oplus}} \right)$$

$$\frac{G M_{\oplus}}{R_{\oplus}^2}$$



$$g_m \left(1 - \frac{2y}{R_{\oplus}} \right)$$

Tangible: What springs to mind?



$$\Delta L \propto \Delta F$$

$$F_s = F_g$$



$$\Delta \vec{p} = \vec{0} = \vec{F}_{\text{net}} \Delta t$$
$$\Rightarrow \vec{F}_{\text{net}} = \vec{0}$$

Interactions

Vectors and scalars

Δ

Definition of velocity

Position update formula

Definition of momentum

$$\vec{p} = \gamma m \vec{v}, \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \quad \text{if } v \ll c \text{ then } \vec{p} \approx m \vec{v}$$

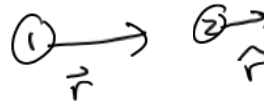
Momentum principle

$$\begin{aligned} \Delta \vec{p} &= \vec{F}_{\text{net}} \Delta t \\ \vec{p}_f &= \vec{p}_i + \vec{F}_{\text{net}} \Delta t \end{aligned}$$

For multiparticle system

$$\begin{aligned} \vec{p}_{\text{tot}} &= \vec{p}_1 + \vec{p}_2 + \dots \\ \Delta \vec{p}_{\text{tot}} &= \vec{F}_{\text{net, ext}} \Delta t \end{aligned}$$

Conservation of momentum



$$\Delta \vec{P}_{\text{system}} + \Delta \vec{P}_{\text{surroundings}} = 0$$

Gravitational force law

$$\vec{F}_{\text{grav on 2 by 1}} = - \frac{G m_1 m_2}{|\vec{r}|^2} \hat{r}, \quad \vec{r} = \vec{r}_2 - \vec{r}_1, \quad G = 6.7 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

Electric force law

$$\vec{F}_{\text{elec on 2 by 1}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}, \quad \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

Spring force law

$$|\vec{F}_{\text{spring}}| = k_s |s|$$

$$s = |L - L_0|$$

direction is back towards equilibrium

Reciprocity

$$\vec{F}_{\text{on 1 by 2}} = - \vec{F}_{\text{on 2 by 1}}$$

Clicker Questions

Q1 A proton is at location $\langle 0, 3, -2 \rangle$ m. An electron is at location $\langle -1, 0, -6 \rangle$ m. What is the relative position vector from the proton to the electron?

A) $\langle -1, 3, -8 \rangle$ m

B) $\langle -1, -3, -4 \rangle$ m

C) $\langle 1, 3, 4 \rangle$ m

D) $\langle 1, -3, 8 \rangle$ m

E) $\langle 1, 0, 6 \rangle$ m

$$\Delta \vec{r} = \vec{r}_{\text{elec}} - \vec{r}_{\text{prot}}$$

$$= \langle -1, 0, -6 \rangle \text{ m} - \langle 0, 3, -2 \rangle \text{ m}$$

$$= \langle -1, -3, -4 \rangle \text{ m}$$

Q2 Consider a bee flying in a straight line with constant speed. At time = 15 s after 9:00 AM, the bee's position vector was $\langle 2, 4, 0 \rangle$ m. At time = 15.5 s after 9:00 AM, the bee's position vector was $\langle 3, 3.5, 0 \rangle$ m. What is the velocity of the bee?

A) $\langle 0.129, -0.065, 0 \rangle$ m/s

B) $\langle 0.133, -0.067, 0 \rangle$ m/s

C) 2.236 m/s

D) $\langle 0.500, -0.250, 0 \rangle$ m/s

E) $\langle 2.000, -1.000, 0 \rangle$ m/s

Speed

not
velocity

speed is
magnitude
velocity

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

$$= \frac{\langle 3, 3.5, 0 \rangle \text{ m} - \langle 2, 4, 0 \rangle \text{ m}}{15.5 \text{ s} - 15 \text{ s}}$$

$$= \frac{\langle 1, -0.5, 0 \rangle \text{ m}}{0.5 \text{ s}}$$

Q3 At time 12.18 s after 1:30 PM a ball's position vector is $\langle 20, 8, -12 \rangle$ m, and the ball's velocity is $\langle 9, -4, 6 \rangle$ m/s. At time 12.21 s after 1:30 PM, what is the (vector) position of the ball, assuming that the velocity hardly changes in this short time interval?

- A) 24.75 m
- B) $\langle 20.27, 7.88, -11.82 \rangle$ m
- C) $\langle 0.27, -0.12, 0.18 \rangle$ m
- D) $\langle 129.62, -40.72, 61.08 \rangle$ m
- E) $\langle 129.89, -40.84, 61.26 \rangle$ m

$$\vec{r}_f = \vec{r}_i + \vec{v}_{ave} \Delta t$$

Q4

Which of these equations correctly relates the average velocity of an object to the object's initial and final positions?

(That is, which could you use to predict the final position of an object if you knew its initial position and average velocity?)

A) $\vec{v}_{\text{avg}} = (\vec{r}_f - \vec{r}_i) \Delta t$

B) $\vec{r}_i = \vec{r}_f + \vec{v}_{\text{avg}} (t_f - t_i)$

C) $\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}} (t_f - t_i)$

D) $\vec{r}_f = \vec{r}_i + \frac{\vec{v}_{\text{avg}}}{(t_f - t_i)}$

E) $\vec{r}_f = \vec{v}_{\text{avg}} t_f$

Q5

<p>Which of the following can NOT be true for an object moving in a straight line at a constant speed?</p>	<p>A. Nothing is interacting with the object (it is in interstellar space, far from all other objects).</p> <p>B. The object is experiencing a net interaction.</p> <p>C. The object is experiencing multiple interactions, and these interactions add up to zero.</p> <p>D. The object has no net interaction with the rest of the world.</p>
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Q6

Three protons travel through space at three different speeds.

Proton A: 290 m/s

Proton B: 2.9e6 m/s

Proton C: 2.9e8 m/s

For which proton(s) is it reasonable to use the approximation $\gamma \approx 1$ when calculating its momentum?

A) A only

B) A and B

C) A and B and C

D) none of the protons

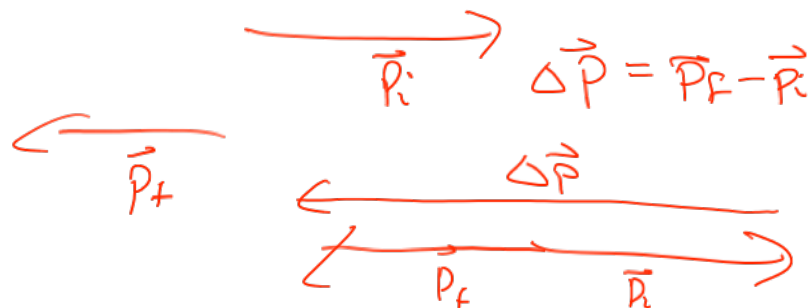
Q7

Suppose you are driving a 1000 kg car at 20 m/s in the +x direction. After making a 180 degree turn, you drive the car at 20 m/s in the -x (opposite) direction. What is the **magnitude of the change of the momentum** of the car $|\Delta \vec{p}|$?

- A) 0 kg·m/s
- B) 2.0e4 kg·m/s
- C) 4.0e4 kg·m/s
- D) 6.0e4 kg·m/s
- E) 8.0e4 kg·m/s

$$(1000)(-20) \frac{\text{kg} \cdot \text{m}}{\text{s}} - (1000)(+20) \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

$$= -40000 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$



Q8

Suppose you are driving a 1000 kg car at 20 m/s in the +x direction. After making a 180 degree turn, you drive the car at 20 m/s in the -x (opposite) direction. What is the **change of the magnitude of the momentum** of the car $\Delta|\vec{p}|$?

- A) 0 kg·m/s
- B) 2.0e4 kg·m/s
- C) 4.0e4 kg·m/s
- D) 6.0e4 kg·m/s
- E) 8.0e4 kg·m/s

$$\begin{aligned} |\vec{p}_i| &= 1000 \text{ kg} \times 20 \text{ m/s} = 20000 \frac{\text{kg} \cdot \text{m}}{\text{s}} \\ &= |\vec{p}_f| \\ \Delta |\vec{p}| &= 0 \end{aligned}$$

Q9) A spring has a length of 8 cm when relaxed, and a length of 10 cm when stretched by a force of 40 N. What is the stiffness k_s of this spring?

A) 4 N/m

B) 5 N/m

C) 20 N/m

D) 400 N/m

E) 2000 N/m

Q10) A spring has a length of 8 cm when relaxed, and a length of 10 cm when stretched by a force of 40 N, so its stiffness k_s is 2000 N/m. The spring is compressed to a length of 7 cm by an unknown force. What is the magnitude of this force?

- A) 20 N
- B) 70 N
- C) 140 N
- D) 7000 N
- E) 14000 N

$$\begin{aligned} F_s &= k_s |s| \\ &= 2000 \frac{\text{N}}{\text{m}} |.08 - .07| \text{m} \\ &= 20 \text{ N} \end{aligned}$$